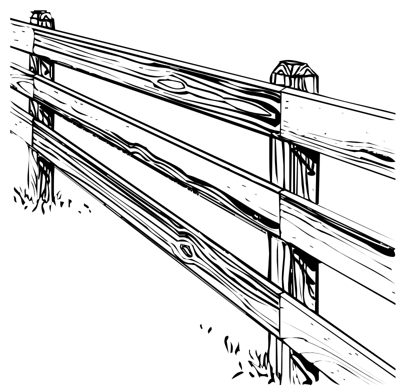
Activity 2: Old MacDonald's Pigpen

Math Concepts:	**Materials:**
• Scatter plot • Quadratic Function • Maximum value of a parabola	• TI-84 Plus

Overview

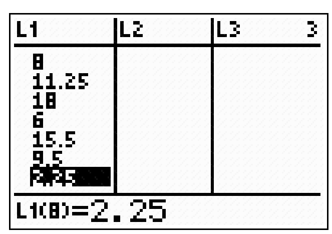


Old MacDonald has 40m of fencing to make a rectangular pen for his pigs. If he wants to give the little porkers as much room as possible, what should the dimensions of the pen be?

This simple version of a standard maximum value problem invites the use of extra features of the calculator to save time and ensure accuracy. This problem is easily adapted for pre-algebra, algebra, or even pre-calculus. Pre-algebra students can comprehend the idea that the area varies as the width and it is likely there is one rectangle with the greatest area and that the calculator can identify that one rectangle for them. Students who have already studied parabolas and their quadratic equations can match the mathematical theory to the calculator's answers.

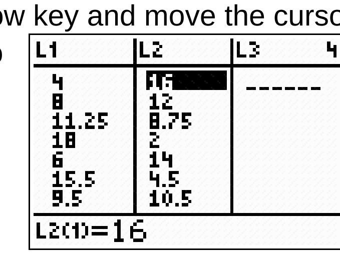
Setup & Data Collection

1. Allow students to work in groups to fill in the chart on their worksheet. Then guide them in a discussion toward finding a general formula for finding the length. If 2 of the lengths plus 2 of the widths is forty, for example, ask them what one of the widths plus one of the lengths would be? If L+ W = 20 then L = 20 – W.



2. Have the students enter the widths of the rectangle in **L1**. *(For help with entering data into lists, see Appendix B)*

3. Rather than having to type in all the lengths, show the students how you can let the calculator do the work for you by defining **L2** with the formula you arrived at in your discussion. Use the up arrow key and move the cursor on top of the **L2** symbol. Now type **"20- L1"**. Press a + to access the quotation marks and ` 1 to display **L1**.



Note: Enclosing the formula in quotation marks will

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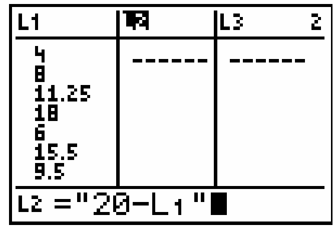
automatically upgrade **L2** whenever you make a change to **L1**.



4. Whenever you have the name of a list highlighted, the command you enter will be applied to the entire list. What you type will appear at the bottom of the screen and when you press e the list will be filled in. (COOL!)

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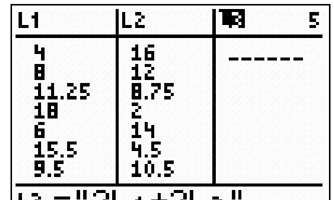
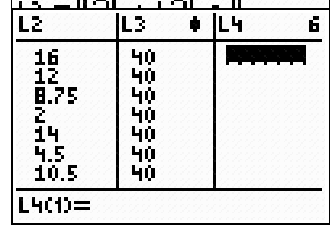
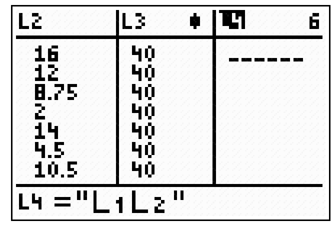
5. Let the calculator also find the perimeter and area for you. With the cursor highlighting **L3**, type **"2L1+2L2"**. The keystrokes would be a + 2 ` 1 + 2 ` 2 a +.



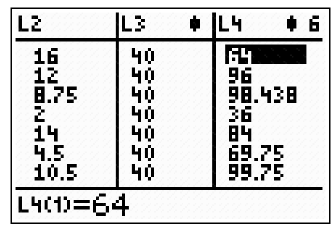
6. .Press e. This will confirm that each rectangle is using all 40 meters of fencing.



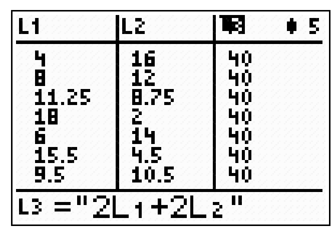
7. Now highlight **L4** and type **"L1L2"**. The keystrokes are a + ` 1 ` 2 a +.



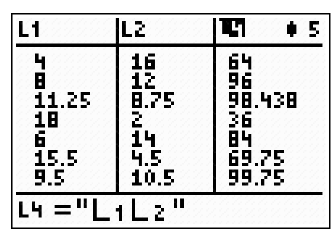
8. Press e to see the areas of the rectangles fill in **L4**.



9. Since it is clear that all the perimeters really are the same, you don't really need **L3**. You can only see three lists at a time, and you want to see **L1, L2, and L4**. You can control the order in which lists appear on your screen. Simply put the cursor on **L3** and press the d key.



10. This will delete the list from being shown in the display but it does not erase the values in the list. Now you can see the three lists you are using for the rest of this investigation. *(For help with adjusting the display of lists, see Appendix B.)*



11. As you look at the lists you begin to wish you had put the widths in order when you first entered them and then maybe you would have been able to more readily identify a pattern.

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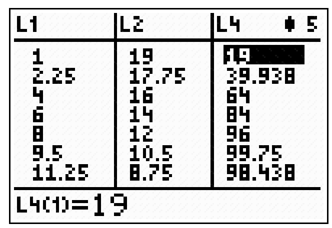
12. No problem! Let the calculator sort the list for you. Press the S key and select **2:SortA(** from the menu. This will sort the list in ascending order.

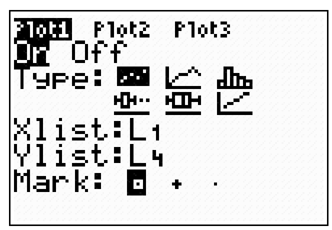
13. You will be on the home screen. If you enter **L1**, the calculator will arrange the numbers in **L1** in order but, unless the other lists were built with quotation marks, it will leave the numbers in the other lists alone. Because the numbers in the other lists here are related to the numbers in the first list, you want the entire row to be carried along with the lead entry from **L1**. To do this simply type in **SortA**(**L1**). Because **L2** and **L4** were created with quotation marks, their entries will follow the entries in **L1**. You would need to type **SortA(L1, L2, L4)** when working with lists were built without quotation marks. Press e to execute the command. The calculator has now accomplished what you wanted.

14. Go to S, **1:Edit…**Your data has been sorted. Notice that **L1** is listed in ascending order and its corresponding length and area have remained in line with it.

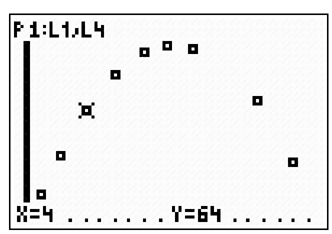


Data Analysis

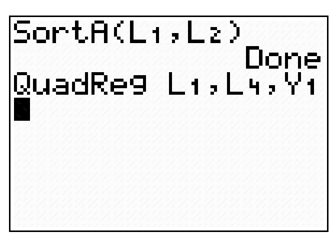
1. Now press ` ! to access å and create a scatter plot. *(For help setting up a scatter plot, see Appendix F.)* Use **L1** for the **Xlist** and **L4** for the **Ylist**. This will be graphing the area, **L4**, as a function of the width of the rectangles, **L1**. This is a good place to review vocabulary such as Y is a function of X; area as a function of the widths. Help students identify the width as the independent variable and the area as the dependent variable.



2. Press #. Scroll down to highlight **9:ZoomStat**, and press e, or simply press 9. You should see your points plotted. Press $ and scroll right and left to see the **X**- and **Y**-values of the data points. Ask students to name the shape for the graph if the points were connected. Lead them to realize it looks like a parabola and the calculator can find its equation for them.



3. Find the regression equation and paste it in **Y1**. Press S and scroll over to **CALC** and down to **5:QuadReg**. Press e. When **QuadReg** appears on the home screen type **L1**, **L4**, **Y1** after it. This tells the calculator which lists to use



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for the **X**- and **Y**-values where to paste the regression equation. The keystroke sequence is as follows: ` 1 , ` 4 , v >. Choose **1:Function** and then **1:Y1**. Press e. (` 1 is used to access **L1** and ` 4 will access **L4**.) *(For more help with finding regression equations, see Appendix G.)*

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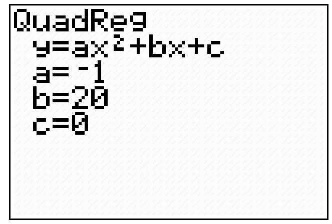
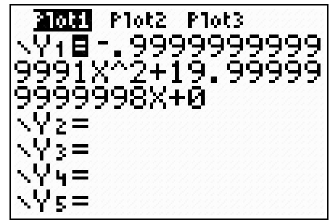
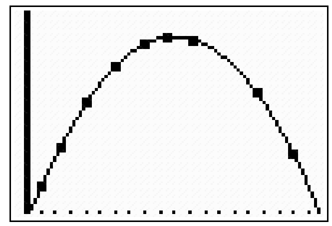
4. Press e. The regression equation will be displayed on the home screen.

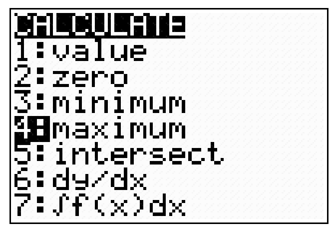


5. Go to the ! window. Look at those funny numbers! Although the **a** value on the home screen was negative one, here it has a rounding error in it, as does the **b** value. This is a great time for a discussion about the fact that the calculator is a tool and is only as good as the person using it. Students need to recognize the -99999999 as equivalent to -1.

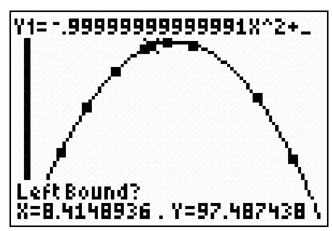




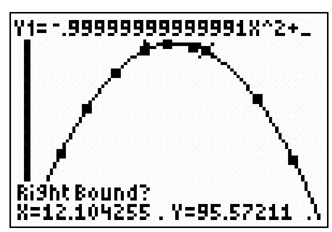
6. Press % and watch the calculator play dot to dot with the points that were already plotted. It's a real crowd pleaser! Examine how closely the regression equation fits the points.



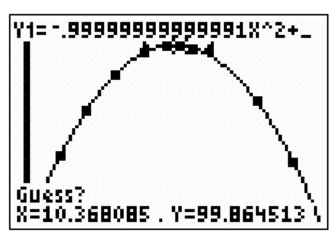
7. Time to find the maximum area. Press ` $, to access the è menu, and arrow down to **4:maximum**. Press e.



8. You will be taken to the graph. Follow the onscreen directions to identify the maximum area. When asked for the **Left Bound?**, move the left arrow key until the cursor is clearly to the left of the vertex and press e. Your choice is marked and the onscreen question changes.



9. When asked for the **Right Bound?**, move the right arrow key until the cursor is clearly to the right of the vertex and press e. Again, your choice is marked and the onscreen question changes.



10. When asked for a **Guess?**, move the cursor close to the vertex point and press e.

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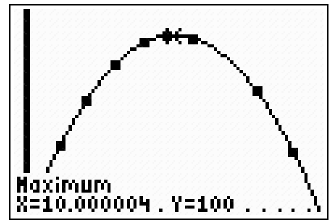
11. Ah! More funny numbers! More talk!



Discussion Notes

I like this problem because it allows me to use many of the features of the calculator. To me it emphasizes that the calculator is a tool, not a cure all, and is only as good as the person using it. Teacher attitude adjustment time (if needed): Did I need the calculator to do this problem? Of course not! Could I use this problem to increase my students' knowledge of the capabilities of the calculator? Absolutely!!!

Guide your students in a discussion that will lead them to an understanding of which variables stand for which values in this problem. You should be able to count on students at this level knowing that a rectangle's Area = length X width. Help them see this problem is that simple. Substituting x for the width, and 20-x for the length, you want them to see that the Area = (20 – x) x, or $A(x) = 20x - x^2$. In standard form $A(x) = -x^2 + 20x$. Matching this to the standard $y = ax^2 + bx + c$, a = -1, b = 20, and c = 0. These are definites and help to conclude that those funny looking numbers in the ! screen are just rounding errors. Pre-algebra students will need to depend on the calculator to identify the maximum but algebra students who have studied quadratics can identify the **X-**value of the vertex as being found at -b/2a. For the values in this problem, that would be -(20)/2(-1) = 10. This is the width of the rectangle with the largest area. The area would be 10(20-10) = 100. The worksheet is designed to help lead students through this same reasoning.

Worksheet Answers

1-10. See chart

11. A(x) = x (20-x),
12. quadratic, parabola
13. vertex
14. width
15. area
16. width
17. maximum area

Rect #	Width	Length	Per	Area
1.	4 m	16	40	64
2.	8 m	12	40	96
3.	11 ¼ m	8.75	40	98.438
4.	18 m	2	40	36
5.	6 m	14	40	84
6.	15 ½ m	4.5	40	69.75
7.	9 ½ m	10.5	40	99.75
8.	2 ¼ m	17.75	40	39.983
9.	1 m	19	40	19
10.	x	20-x	40	x (20-x)

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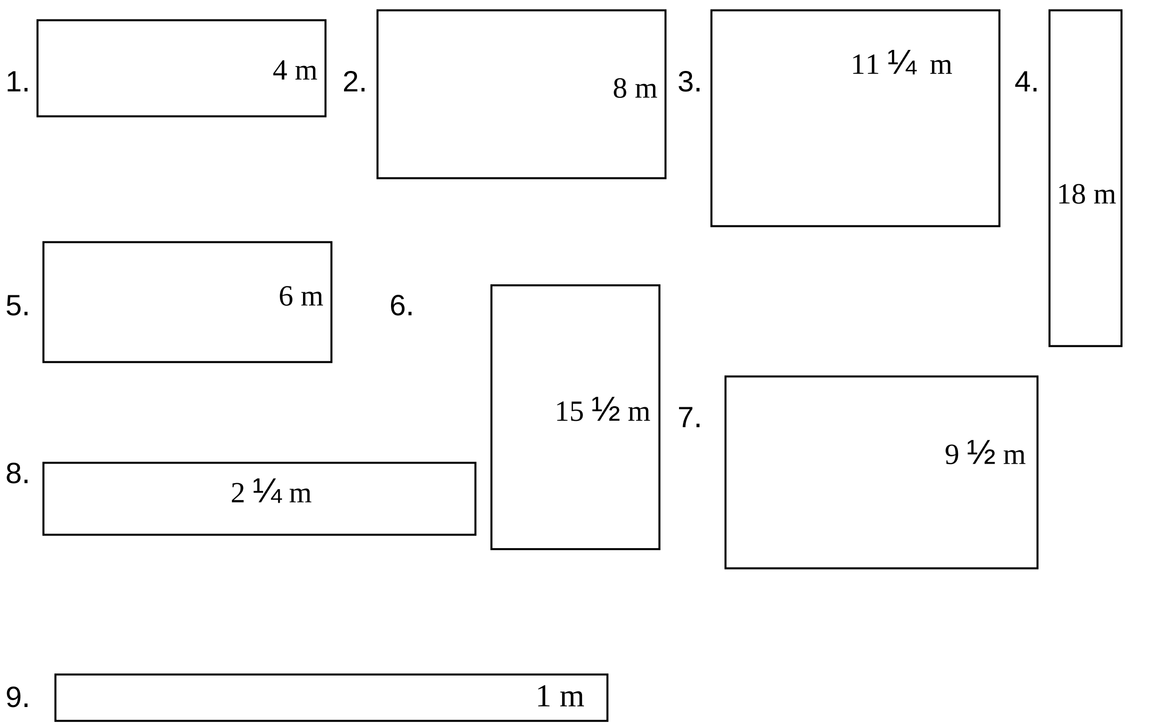
Old MacDonald's Pigpen Worksheet   Name____________________________

Old MacDonald has 40m of fencing to make a rectangular pen for his pigs. If he wants to give the little porkers as much room as possible, what should the dimensions of the pen be? He started to figure this out by drawing a few sample rectangles below. Mark the sides of the following rectangles to ensure that each rectangle has a perimeter of 40m. Fill in the chart below making sure the perimeter is always 40 m. Find the area of each rectangle.



10.

Rectangle #	Width	Length	Perimeter	Area
1.	4 m			
2.	8 m			
3.	11 ¼ m			
4.	18 m			
5.	6 m			
6.	15 ½ m			
7.	9 ½ m			
8.	2 ¼ m			
9.	1 m			
10.	x			

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11. It seems obvious that the area of the rectangle is a function of the length of the sides. Use **X** to denote the width and write an equation that relates the area to **X**. This is no harder than saying Area = length X width. A(x)= ________________.
12. This is a _____________ equation. Its graph will be a _____________.
13. If we wanted to find the maximum area, all we would have to do is find the _____________ of the graph.
14. The **X**-value stands for the _______________ of the rectangles.
15. The **Y**-value stands for the _______________ of the rectangles.
16. The **X**-value of the vertex tells us the _____________ of the rectangle with the maximum area.
17. The **Y**-value of the vertex tells us the value of the _________ _________.

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