Name ____________________

Date ____________________

EXPLORATIONS

Activity 10

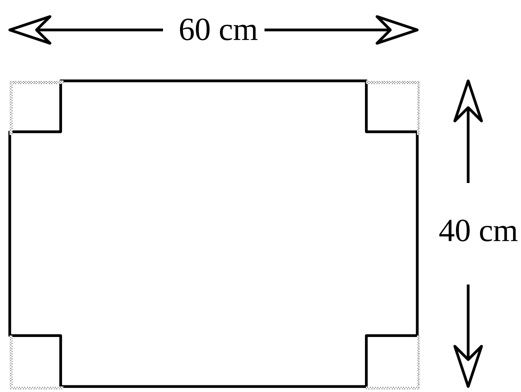
Box It Up (A Graphical Look)

This activity is an extension of *Activity 9: Box It Up*. You saw that cutting out squares with lengths of whole centimeters did not produce a box with the largest possible volume. The best value for the size of the square was between 7 and 8 cm.

In this activity, you will graph the relationship between the length of the sides of the cut-out squares and the volume of the resulting box.

The Problem

Ms. Hawkins, the physical sciences teacher at Hinthe Middle School, needs several open-topped boxes for storing laboratory materials. She has given the industrial technologies class several pieces of metal sheeting to make the boxes. Each of the metal pieces is a rectangle measuring 40 cm by 60 cm. The class plans to make the boxes by cutting equal-sized squares from each corner of a metal sheet, bending up the sides, and welding the edges.



In Activity 9, you built a table showing the volumes that resulted when squares of certain sizes were removed. Another approach often used to solve such problems is to generate a graph of the relationship between box height and volume on a coordinate grid.

You may want to spend some time introducing the terms "coordinate grid" and "coordinate plane" if this terminology is new to your students.

Using the Calculator

To graph a relationship on a graphing calculator, it is necessary to carry out two tasks:

a. Define the relationship in terms of two variables, x and y.

b. Define the portion of the coordinate plane over which you wish to view the graph.

1. If you have not cleared your [Y=] screen from *Activity 9*, you should still have **Y1** defined as $(60–2x)(40–2x)x$. If not, press [(] 60 [–] 2 [X,T,Θ] [)] [(] 40 [–] 2 [X,T,Θ] [)] [X,T,Θ]. Once finished, you have completed the first of the two tasks necessary for graphing a function.

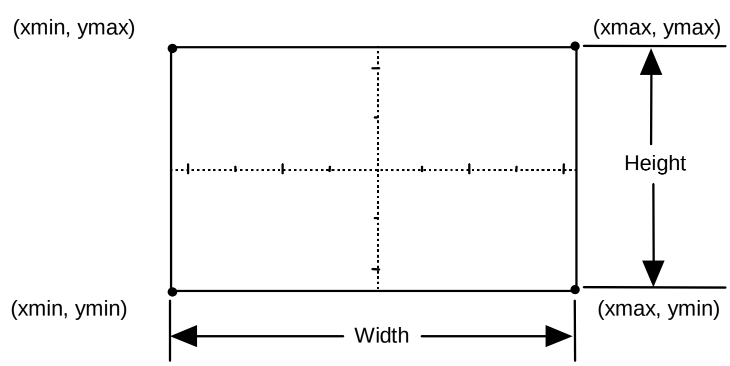
Defining the limits for the viewing windows for graphs provides students much needed practice in thinking about the function and its connection to the problem situation. They need to ask themselves "What input values make sense for this problem?" (a restricted domain) and "What output values do I get with those inputs?" (the range). When drawing graphs by hand, all too often students will just draw the two perpendicular axes, make tick marks, each 1 unit apart, and then find corresponding y-values to x= 0, 1, 2, and so on, even if it doesn't make sense to use those values for the problem situation.

The second task is to define the portion of the coordinate plane that you wish to view. It is common to refer to this as the *viewing window* and the *limits* of this view must be defined properly before you have the calculator graph the function. The screen used to define the limits of the viewing window is accessed by pressing the [WINDOW] key.

2. Press the [WINDOW] key now. You should see a screen that allows you to define six values: **Xmin**, **Xmax**, **Xscl**, **Ymin**, **Ymax**, and **Yscl**.

You may also see a line for defining **Xres** on some calculators. This value can remain at 1 for purposes of this activity.

As illustrated below, the values of **Xmin** and **Xmax** define the left and right endpoints of the viewing window and the values of **Ymin** and **Ymax** define the upper and lower limits of the viewing window.



Viewing Window

The values of **Xscl** and **Yscl** (abbreviations for *x-scale* and *y-scale*) have no effect on the window limits nor on the appearance of the graph. They are used to define the distance between reference (or *tick*) marks that will appear on the two axes or along the left and bottom edges of any generated graph. Later in the activity when you graph the function, you can change the values for **Xscl** and **Yscl** to verify this for yourself and to help you understand why changing the scale does not change the appearance of the graph.

*It will be worth the time to let some students explore what happens when **Xscl** is changed. Some will think the graph should shrink or enlarge. Let them think about why this does not happen. Avoid the temptation to just tell them. Suggest that they think of a ruler. Does the distance between decimeters change when centimeter or millimeters marks are added?*

Now define the limiting values for the viewing window. To define the limits on x, consider the equation

$$Y1 = (60-2x)(40-2x)x$$

that provides the computed volumes and think about how the equation is used to answer our original problem. Given the conditions from the problem situation, x must take on values between 0 and 20.

✐ Go to the **Questions** section and answer #1.

It makes sense in this problem to define **Xmin** to be 0 (or slightly less than 0 to allow for viewing near 0) and **Xmax** at 20 (or slightly more than 20 to allow for viewing near 20). If tick marks are desired, you can set **Xscl** to 1 and have tick marks at every unit along the x-axis.

Defining the limits on y is a little more challenging. A given value of **Y1** represents the volume of a box for some height x. Volume cannot be negative, so you can define **Ymin** to be slightly less than 0. You are looking for the x that would produce the largest value of y possible. This doesn't give much of a hint about how large y can get. However, your previous work with tables showed that all volumes were less than 8500 cubic centimeters, so a reasonable value for **Ymax** is 9000. Since the height of this view is very large, the **Yscl** should be large. Enter 500 for **Yscl** so that tick marks on the y-axis are placed at 500, 1000, 1500, … , 9000.

This shows the students that they need to consider particular values of the function when sketching graphs by hand also.

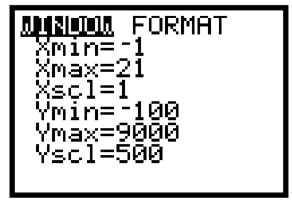
Allow the students to reason and explain why there are many reasonable values. Number sense can be developed as students think about the choices for reasonable values for scaling the axes.

Since this may be the first time students are working with creating windows, specific values are provided.

3. Based on this analysis, enter the following values for the six variables in the **WINDOW** screen. Enter them into your calculator *exactly* as shown. Use [▼] and the numeric keys to enter the values on the corresponding lines.

Xmin = –1	**Xmax = 21**	**Xscl = 1**
Ymin = –100	**Ymax = 9000**	**Yscl = 500**

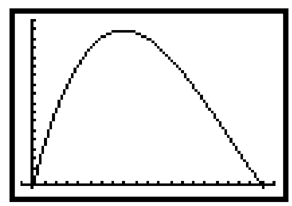
When entering the negative values, be certain to use the [(-)] key and *not* the [−] key. Your final screen should look something like the one shown at the right.



Since the tasks of defining the expression and setting the window limits is completed, it is time to view the graph.

Starting with x=0, the volumes increased (fairly quickly) up to a maximum volume between x=7 and x=8. The volumes then decreased beyond x=8. The graph shows this pattern of increasing and decreasing also with the maximum occurring between x=7 and x=8.

4. Press the [GRAPH] key and wait for the *working indicator* in the upper right corner of the display to disappear. A graph similar to the one at the right should appear on your screen.



✎ Go to the **Questions** section and answer #2.

You may want to ask students why the graph on the calculator is "flat" at the top. Are there several x-values that produce a maximum volume?

5. Press the [TRACE] key once to read coordinates of points on this graph.

Note that the blinking crosshair and the coordinates are displayed at the bottom of the screen. This informs you that the point where $x = 10$ and $y = 8000$ lies on this view of the graph.

✎ Go to the **Questions** section and answer #3.

It is possible to read other points that lie on this view of the graph.

6. Use [◄] and [►] to move the blinking crosshair. The crosshair will move only along the graph and not just anywhere in the coordinate plane.

Notice that the coordinates at the bottom of the screen change as you press the arrow keys. The *x*-coordinates are determined by the calculator and are based upon the current values of **Xmin** and **Xmax**. The *y*-coordinates are computed using the expression $(60–2x)(40–2x)x$. You will probably see that the *x*-values are not always "nice" numbers like those you saw when you generated the tables. This sometimes makes it more difficult to decide on solutions to real world problems being modeled by the graphs.

7. Move the cursor until you locate the point on the graph that gives the *largest y-coordinate.*

The x value of 10 cm indicates the side lengths in centimeters of the cutout square and the y value of 8000 cm³ is the volume of the corresponding box.

Ask why not all of the decimal places are meaningful.

✐ Go to the **Questions** section and answer #4.

Not all of the decimal places shown in the coordinates of this point are meaningful. In fact, this point may not be the one you are looking for. It is just the closest point on this view of the graph. By examining the *x*-coordinates of the points on *each* side of this point, you can find an interval that contains the best value of *x*.

8. Use the arrow keys to determine the *x*-coordinates of the points immediately to the left and to the right of the point found above.

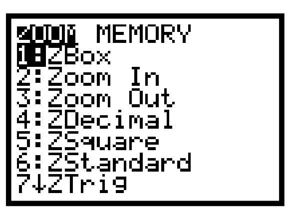
✐ Go to the **Questions** section and answer #5.

The solution you are looking for lies somewhere between these two *x*-values. In *Activity 9*, you found solutions by generating tables of values. Check to see that these tabular solutions fall somewhere between the two values you have written above.

*(**Hint**: Press* [2nd] [TABLE] *to access the table of numbers.)*

Just as we did when we used tables to investigate this problem, it is easy to examine the volume function over a smaller interval in order to obtain a more precise approximation to the desired solution. This can be accomplished a number of ways, but the fastest is to use the [ZOOM] capabilities of the calculator.

9. Press the [ZOOM] key and examine the menu on the screen. Two of the options shown are **Zoom In** and **Zoom Out**.



On some calculators, it is possible to save the current graph prior to zooming. That way if the students are not satisfied with their "zoom", they can easily start over again. Once they press the [ZOOM] *key, have them arrow over to **MEMORY** and select the **ZoomSto** option. Then press* [ZOOM] *again and continue with the instructions given. Whenever they want to return to the stored zoom, they need to press* [ZOOM]*, **MEMORY**, and select **ZoomRcl**. On the TI-82 and TI-83, the **Zprevious** option also allows students to recall the previous zoom.*

Zooming changes the limits of the viewing window. *Zooming in* is similar to bringing a portion of your current view closer to you for a finer examination—much like looking through a pair of binoculars. *Zooming out* is somewhat equivalent to enlarging your field of view so that you see more but in less detail.

Since you need a closer, more detailed look at the points near the apparent maximum of the graph, you must zoom in around the region containing that maximum. A quick way to zoom in is to use the option named **ZBox**. **ZBox** lets you use the cursor to select opposite corners of a box to define the portion of the current view that you wish to enlarge.

10. Press **1:ZBox** to select the **ZBox** option from the **ZOOM** menu.

You will see a different crosshair in the center of the screen.

Envision creating a small box that contains the highest point of the graph. There are many such boxes, and it doesn't really matter if yours is the same as anyone else's. You are going to move the cursor to the *upper left corner* of that box.

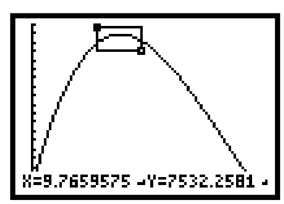
When students hold down the arrow key as they create the box, the box disappears until they stop pressing the key.

11. Use the blue arrow keys to move the cursor somewhere above and to the left of the highest point of the graph.

12. Once you are satisfied with the cursor's location, press the [ENTER] key. You have now selected one corner of the zoom box.

13. Now move the crosshair to the right and then down until you reach the opposite corner of the box you are forming. As you do this, you will see the box forming on the screen.

Try to center the point of interest in the box. One of many possible zoom boxes is shown at the right.



14. When you are satisfied with the location of the second corner, press [ENTER] again.

The calculator should redraw the graph in the window you have just defined.

15. Use [TRACE] and the arrow keys again to locate where you believe the maximum value to be.

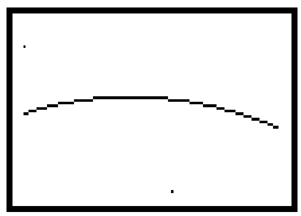
✎ Go to the **Questions** section and answer #6 and #7.

If the tenths position in the *x*-interval is not the same for both values, or if both *x*-values when rounded to the nearest tenth are not the same, then the best choice for *x* we can make has to be a whole number. If you zoom one more time on the place where you believe the maximum volume will be, you will be able to provide a value for *x* that is accurate to the nearest tenth.

16. Press [ZOOM] and select **Zbox** again. Make another box around the area on the graph where you think the maximum volume lies.

17. Once you've made the box, press [ENTER].

A new graph appears *perhaps* looking something like the graph displayed. Remember, we haven't necessarily chosen the identical *zoom box*.



18. Press [TRACE] to find new *x*-values for an interval containing the maximum volume as you have done previously.

✎ Go to the **Questions** section and answer #8 through #11.

A tiny crosshair for the ***Zbox*** *is still showing on the screen. This allows you to continue creating "zoom boxes". This is not the trace key crosshair. Students still need to press* [TRACE] *again . Students are now learning a bit about false precision and accuracy. To ensure that the x value chosen is accurate to the nearest tenth, we need to find an interval of values where the tenths digits are the same when any of the x values are rounded to the nearest tenth.*

Notice how the graph appears to be getting flatter and flatter as one zooms in on the maximum value. Several interesting "upper level" mathematical ideas can spring from this investigation: the fact that every graph is "locally linear", that is, that all graphs will appear to be a straight line as we zoom in on smaller and smaller intervals of x, and that the slope of the tangent line at the local maxima and minima of a function is zero.

Remind the students that they cannot simply read off the volume from the graph as the volume is a computed value based on the x-value given. They need to compute the volume based on their chosen value for x.

Questions

Because 1 centimeter is the smallest whole centimeter cut and if more than 19 centimeters is cut, the box would have no volume (one of its dimensions becomes less than or equal to zero.)

1. Why must *x* have a value between 0 and 20?

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Return to page 99.

2. In your group, write a brief explanation of why the graph should look the way it does based on your previous work with the volume table and the patterns you observed there.

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Return to page 100, step 5.

If the students have keyed in the correct window values on the TI82/83, the crosshair should be on the point indicated. If students have just pressed the right or left arrow, their cursor will not appear on the graph. Have them push the TRACE *key. For the TI-80, the cursor will appear elsewhere due to the size of the graphing screen.*

3. Do the coordinates of points on the graph agree with the values you computed earlier in this activity? Explain what these two values mean with respect to the box volume problem.

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Return to page 100, step 6.

x=7.893617
y=8450.2224

4. What are the values of *x* and *y* at the highest point on the graph?

x= ________________ *y*= ________________

Return to page 101, step 7.

x left = 7.6595745
x right = 8.1276596

5. Record the *x*-coordinates from page 101.

Point to left: ________________

Point to right:________________

Return to page 101, step 8.

Sample values:
x=7.8388411,
y=8450.4392.

6. Record the coordinates from step 15 on page 103.

x= ________________ *y*= ________________

Return to page 103, step 15.

7. By observing the *x*-coordinates of the points on each side of this point, you can again determine an interval that contains the desired solution. What *x*-interval contains the value associated with the maximum height of the graph in step 15, page 103?

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Sample x interval: (7.78, 7.89). Values displayed only to the nearest hundredth.

Students may have to do several zooms in order to get values in an x interval whose tenths digits round to the same number. If they zoom in with a tiny box around the maximum value, they may arrive at such an interval sooner, for example (7.846, 7.847).

Return to page 103, step 15.

8. Write the new interval for the maximum volume. Determine whether you can provide an x-value accurate to the nearest tenth. Remember, you want all numbers in your interval to round to the same tenth's value.

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9. Based on the results from your trace of the graph, what size squares should the class cut from the pieces of sheet metal? What are the dimensions and volume of the box with the largest volume?

Side length of square: ________________

Height: _________ Length: ________

Width: _________ Volume: _______

Side length of square: 7.8 cm

H=7.8 cm
L=44.4 cm
W=24.4 cm
V=8450.208 cm^3

10. You have now investigated the box volume problem using two methods: tabular and graphical. Discuss in your groups the advantages of both methods. Write those advantages below.

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Advantages will vary based upon the group discussion. Some students prefer the numerical/tabular approach. The ***ΔTbl*** *feature makes "zooming in" on a table fairly easy. Others prefer a picture and would rather zoom in on the graph for a maximum value. Sometimes when viewing a finite table of values, it is difficult to get a sense of how the function "behaves" over a larger interval; the graph provides a "bigger picture". One can find a maximum value fairly quickly on the table without having much sense about how the function behaves. When creating a graph, you have the additional challenge of defining the window. This requires some knowledge of the behavior of the function.*

11. Which method do you prefer and why?

____
____
____
____
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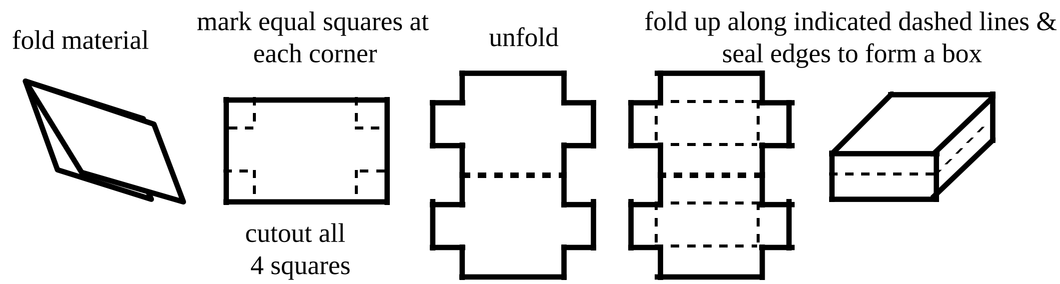
Problems for Additional Exploration

H=11.0 cm
L= 53 cm
W=38 cm
V=22154 cm^3.

1. Assume that the rectangular metal sheets given to the industrial technologies class each measure 75 cm by 60 cm, and that we still want to determine the size of the square to cut out so that a box with the largest volume is produced. Use both tables and graphs to work through a solution to this problem.

2. Instructions for making a suitcase-like box from a rectangular sheet of material are given below.



A grid master is provided in the ***Appendix*** *that can be copied, distributed, cut, and folded to form sample boxes.*

a. Use a piece of graph paper or plain copy paper to make a model of the box. Note that the size of the cut-out square determines the dimensions of the resulting box.

The function for this box problem looks like V= (36-2x)(48-2x)2x. Remember, the box is folded so after you cut the 4 squares from the side of 72 inches, you use half of the remaining length to make the width of the folded box. Hence, the expression (36-2x) versus (72-4x).

b. Assume that the material being used to make such a box measures 48 inches by 72 inches. Determine the dimensions and volume of the box formed when 6-inch squares are cut from the corners of the folded paper.

c. Use the techniques you have learned in this activity to determine the size of square that should be cut to form the box with the largest possible volume.

When x=6,
H=12 cm,
L=36 cm,
W= 24 cm, and
V= 10368 cm^3.

Maximum volume occurs at x=6.8 cm, accurate to nearest tenth, with
H=13.6 cm,
W= 22.4 cm,
L= 34.4 cm,
V= 10479.6 cm^3.